# OUTPUT NOISE ANALYSIS

Tutorial - EN

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*SCA CONTROL - Control systems for your processes*

# 1 Introduction

Output noise (or measurement noise) can generate undesired control actions resulting in wear of actuators and reduced performance. The effects of measurement noise can be alleviated by filtering the measurement signal. However, heavy filtering likely degrades the performance of the controller. Consequently, to decrease the need for filtering, the controller's robustness to measurement noise plays an important role. The aim of this paper is to prove that AC controllers can provide a better robustness to measurement noise than PID controllers, especially for 1p-processes.

# 2 Theoretical background

Suppose that the reference $r(k)$ is constant and the output $y(k)$ has converged to the regime value $\bar{y}$. Then, suppose that $y(k)$ is affected by white noise $n(k)$ with variance $\sigma_y^2$. Therefore, $y(k)$ can be modeled as a WSS (wide-sense stationary) Gaussian Process characterized by the following parameters:

- mean $\mathbb{E}[y(k)] = \bar{y}$
- variance $var[y(k)] = \sigma_y^2$
- statistical power $M_x = \bar{y}^2 + \sigma_y^2$
- signal power $P = \frac{1}{N}\sum_{k=1}^{N} y(k)^2$. It is related to the electrical power, the exact relationship depends on the actuation nature. Using the Law of the Large Number, it is possible to prove that $P$ converges to $M_x$ as $N$ increases.

To compensate the measurement noise, the controller generates a control action which in turn is affected by noise with variance $\sigma_u^2$. To analyze the robustness to measurement noise, the Noise Gain is often defined:

$$k_n = \frac{\sigma_u}{\sigma_y} \tag{1}$$

where the $\sigma_u$ and $\sigma_y$ are the standard deviations. A smaller $k_n$ indicates a greater robustness.

# 3 Experimental setup

Given a particular process structure, both AC and PID controllers are designed for a specific combination of specifications ($\alpha$, $\beta$...). See "preliminaries" document for the explanation of such parameters. Then, white noise with a certain power $\sigma_y^2$ is applied to the output and, for both controllers, the noise gain (say $k_{n,AC}$ and $k_{n,PID}$ respectively) is calculated from a big number of samples (10000). Finally, we get ratio between these two parameters:

$$k_{n,ratio} = \frac{k_{n,PID}}{k_{n,AC}} \tag{2}$$

If $k_{n,ratio} > 1$, it means that the AC controller is more robust to measurement noise than PID for this combination of specifications.

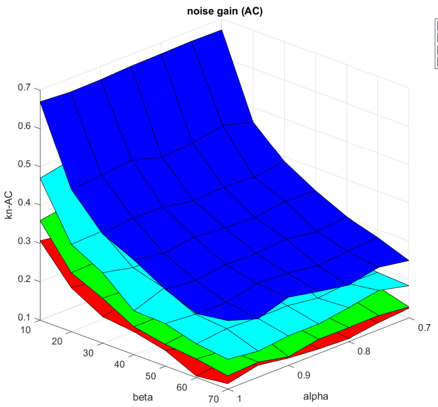
 

The test is then repeated for other values of specifications and all the outcomes are plotted in a graph.

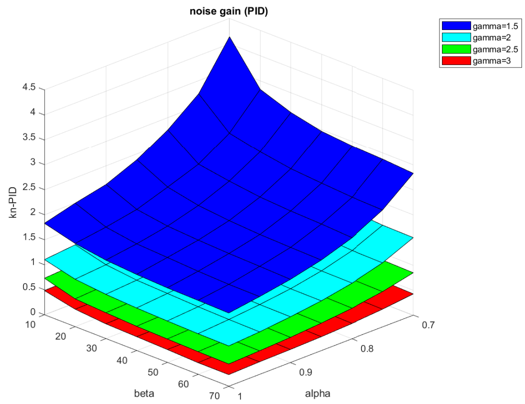
NOTE: it can be proved that $k_{n,ratio}$ does not depend on the noise power or on the process static gain.

## 3.1 1p-processes

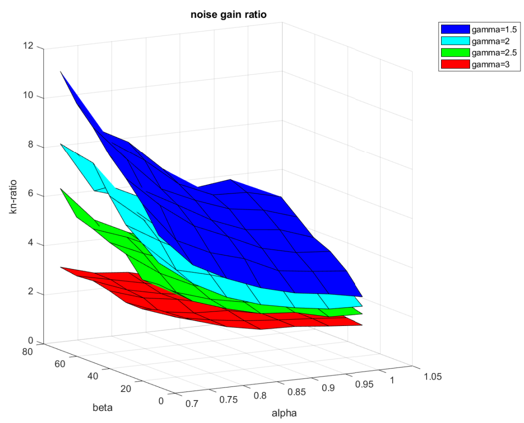
In Fig. 1 and 2, we plot the noise gains and the noise ratio for the 1p-process case with unitary gain. The discrete pole of the process is linked to $\beta$.




(a)




(b)

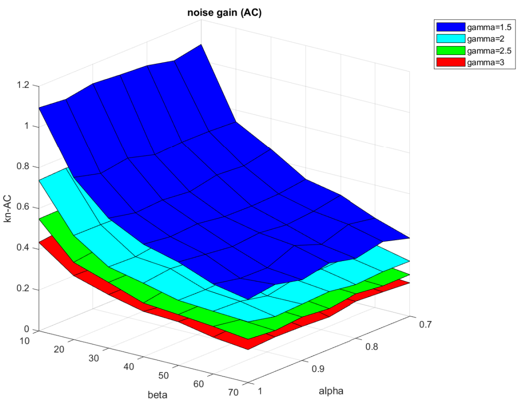



(c)

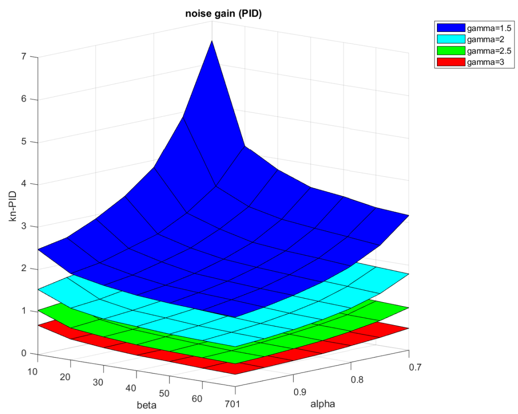
Figure 1: Test for 1p-process with unitary static gain, 10% overshoot, and $\sigma_y^2 = 3e-02$: (a) noise gain with AC, (b) noise gain with PID, (c) noise gain ratio.

As one can observe, both $k_{n,PID}$ and $k_{n,AC}$ increases as $\beta$ and $\gamma$ decrease and as $\alpha$ decreases (only for $k_{n,PID}$). Regarding $k_{n,ratio}$, it is always greater than 1, so the AC is always more robust; this fact is particularly noticeable with lower values of $\alpha$ (higher PID derivative gain), $\beta$ (lower sampling time), and $\gamma$ (lower settling time). Finally, with a lower required overshoot, the noise gains are smaller, while the noise gain ratio is bigger, so the AC is even more robust with a lower overshoot.




(a)

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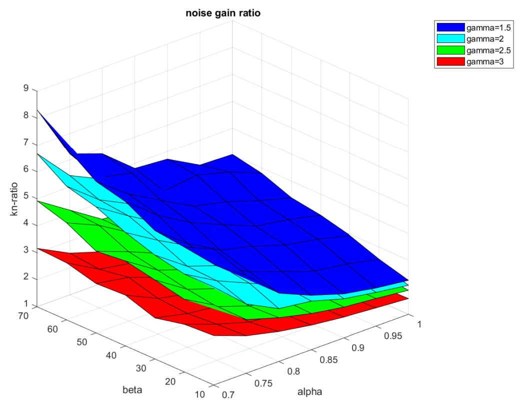


(c)

Figure 2: Test for 1p-process with unitary static gain, 20% overshoot, and $\sigma_y^2 = 3e-02$: (a) noise gain of AC, (b) noise gain of PID, (c) noise gain ratio.

[*results with different structures of the process will be available soon*]

 

# Contacts

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